Parameterization of Fractional-order Proportional Derivative Controller with Wireless Communication System Application

Ali Jasim Ghaffoori
Engineering of Electrical Power Techniques, AL_Ma'moonUniversity College, Baghdad, Iraq
Wameedh Riyadh Abdul-Adheem
Engineering of Medical instrumutation Techniques, AL_Ma'moonUniversity College, Baghdad, Iraq

Abstract

In recent years, derivatives and integrals with fractional order and the structure of conventional proportional-integral-derivative (PID) controllers have been used to construct fractional-order PID controllers for improving the performance of controlled systems. This study focuses on reducing the overshoot for second-order systems with small damping ratio or underdamped systems by using a fractional-order proportional-derivative controller. The proposed design method is direct given the use of a simple formula for selecting a single parameter. A case study based on control the transmitted power of wireless communication network (WCN) to solve the problem, which caused by the large number of data transmissions in wireless network by using construct fractional-order PID controllers to improving the performance of controlled systems. This study focuses on reducing consume power by reducing the overshoot for second-order systems with small damping ratio or underdamped systems.

Keywords: Second-order dynamic systems; wireless communication network; proportional-derivative controller; fractional-order control.

معاملات وحدة تحكم المشتقات النسبية ذات الرتبة الكسرية مع تطبيق نظام الاتصال اللاسلكي

علي جاسم غفوري
كلية المأمون الجامعة/ قسم تقنيات القدرة الكهربائية
وميض رياض عبدالعظيم
كلية المأمون الجامعة/ قسم تقنيات الاجهزة الطبية

الملخص

في السنوات الأخيرة ، تم استخدام المشتقات والتكاملات بالرتبة الكسرية وهيكل وحدات التحكم التقليدية للمشتقات النسبية المتكاملة (PID) لإنشاء وحدات تحكم PID ذات الرتبة الكسرية لتحسين أداء الأنظمة الخاضعة للسيطرة. تركز هذه الدراسة على الحد من

التجاوز لأنظمة الدرجة الثانية ذات نسبة التخميد الصغيرة أو الأنظمة منخفضة التخميد باستخدام وحدة تحكم المشتق النسبي ذات الرتبة الكسرية. طريقة التصميم المقترحة مباشرة بالنظر إلى استخدام صيغة بسيطة لاختيار معامل واحدة. يتضمن هذا البحث دراسة تعتمد على التحكم في الطاقة المرسلة لشبكة الاتصالات اللاسلكية (WCN) لحل المشكلة ، والتي تسببها العدد الكبير من عمليات إرسال البيانات في الشبكة اللاسلكية باستخدام وحدات تحكم PID ذات الرتبة الكسرية لتحسين أداء الأنظمة الخاضعة للسيطرة. تركز هذه الدراسة على تقليل استهلاك الطاقة عن طريق تقليل التجاوز لأنظمة الدرجة الثانية ذات نسبة التخميد الصغيرة أو الأنظمة منخفضة التخميد.

1. Introduction

Many practical dynamics in engineering, such as robot, crane, vibration controls, and power-system electronics, are expressed as second-order systems [1]–[4]. Such systems in their applications may suffer from very high peak values in their measured response curve relative to the required response curve. This situation may lead to the collapse of these systems. Therefore, minimization of the maximum overshoot is required to control such systems optimally [2]. Undesirable vibrations in these systems should also be attenuated in less time.

Proportional-integral-derivative (PID) controllers are the most widely used controllers in control engineering because of their simple structure and limited number of parameters with numerous tuning methods. A PID controller is developed using the Ziegler–Nichols method, wherein the controller parameters are derived from the controlled model while the maximum overshoot is maximized [5]. Employing fractional-order derivatives and integrals leads to a new type of controller called fractional-Order PID (FO-PID) controller.

The FO-PID controller has been receiving interest in recent years as a result of its robust performance [6]. Refs [7], [8] reveal the relationship between the maximum overshoot and the order of differentiation, which motivates this work to solve the parameterization issues of the controller.

2. Literature review and problem statement

Four main fractional-order techniques are considered in literature, namely, tilted proportional and integral (TID), non-integer-order robust control, FO-PID, and fractional lead-lag controllers. The TID controller shares a similar structure with a standard PID controller but replaces the proportional unit with a fractional-order transfer function [9]. The non-integer-order robust control or CRONE, which stands for “CommandeRobusted'Ordre Non Entier” in French, is a preferred control approach-based frequency domain. CRONE established three generations of approaches with many applications [10]. The FO-PID controller is a well-designed and effective fractional-order alteration of classical PID controllers. This controller proposes increased diversity of parameters for the controller. Finally, the integer order of the lead-lag controller is extended by using fractional order [10].

The parameters of an FO proportional derivative (PD) controller can be tuned with genetic algorithm [11], [12], artificial bee colony algorithm [13], particle swarm optimization technique [14], [15], or Nelder–Mead simplex method [16]. The downside of these techniques is the lack of guarantee of optimal results, which may utilize a large amount of processor resources.

The transmitting signals in wireless network systems can detect and process the information in real-time environments, thise can be causes consume power because the effect of many parameters[17]. Wireless networks have a wide range of applications such as, remote areas, cell phone, wireless local area, satellite communication, personal communication, wireless sensor, and microwave networks etc[2]. The power control technology of the wireless network is to reduce the consume power as much as possible without sacrificing the performance of the system, thereby reducing the energy consumption of the improving the survival in real-time of the network and the energy efficiency of the system[18]. Power algorithms in wireless system are considered as making control adjustable. using negative feedback control from receiver to transmitter to get constant power level[19].

3. The aim and objectives of the study

The proposed FO-PD controller with three design parameters is recommended to improve the time response specifications of the conventional PD controller with two design parameters. The proposed controller has been given attention for the same conditions. The simple and direct design procedure considers an extra parameter for the FO-PD controller. The objective of this procedure is to minimize the maximum overshoot. A case study is used to validate the requirements illustrated in this work.

The paper is organized as follows. Section 2 presents the concerned second-order system, description of the FO-PD controller, and theoretical background for the processed method. Section 3 explains the design of the FO-PD controller. Section 4 discusses the case study, which is numerically simulated to prove the efficiency of the proposed design method. Finally, Section 5 presents the concluding remarks.

4. Materials and methods

4.1. Considered Second-order System

The general form of linear systems with second order is as follows [20]:

$$P(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)}, \tag{1}$$

where ω_n is the natural frequency, and ζ is the damping ratio. The characteristic equation of the closed loop with Eq. (1) is the transfer function of the controlled plant for unity feedback and is given as

$$s^2 + 2\zeta\omega_n s + \omega_n^2. \tag{2}$$

The roots of the characteristic equation given in Eq. (2) are

$$-\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}. \tag{3}$$

The second-order system has an underdamped response if the damping ratio is less than 1. In this case, the roots are complex conjugate and described as follows [2]:

$$-\sigma \pm j\,\omega_d, \tag{4}$$

where $\sigma = \zeta\omega_n$, and $\omega_d = \omega_n\sqrt{1-\zeta^2}$ is the underdamped natural frequency.

The transfer function of the open-loop $G(s)$ is constructed by cascading the transfer function of the plant $P(s)$ given in Eq. (1) and the transfer function of the controllers $C(s)$. This arrangement is formulated as

$$G(s)=C(s)\,P(s). \tag{5}$$

Fig. 1 shows a simple feedback loop of the second-order system given in Eq. (1), which is composed of a plant and a controller. r represents the setpoint, e is the control error, u is the control input, and y is the control output [21].

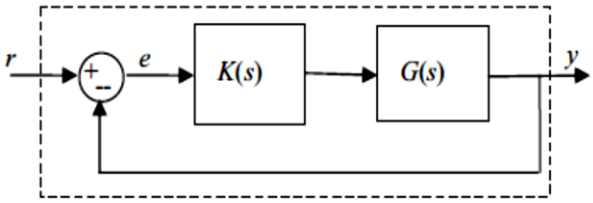

Fig. 1. Plant with a controller for the second-order closed loop [21]

For the underdamped system ($\zeta < 1$), the step response of the uncontrolled system is

$$y(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e_n^{-\zeta\omega_n t} \cos(\omega_d t - \phi). \tag{6}$$

Then, the exact formula for the maximum overshoot value of $y(t)$ is described by the following equation:

$$MP = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}. \tag{7}$$

Moreover, the approximate formula is as follows:

$$MP \cong 1 - \frac{\zeta}{0.6}, \qquad \text{for } 0 \le \zeta \le 0.6\,. \tag{8}$$

4.2. Fractional-order Control Systems and Overshooting Step Responses

The differintegral operator to an arbitrary order is the generalized integer-order differentiation and integration [10]:

$$D_t^{\alpha} = \begin{cases} \frac{d^{\alpha}}{dt^{\alpha}} & \operatorname{Re}\alpha > 0 \\ 1 & \operatorname{Re}\alpha = 0 \\ \int_a^1 (d\tau)^{-\alpha} & \operatorname{Re}\alpha < 0 \end{cases}, \quad (9)$$

where α represents the differintegration order and the a initial conditions are constant.

The differintegral is expressed using the Riemann–Liouville form as [10]

$$D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \quad (10)$$

where $\Gamma(\cdot)$ is the Gamma function. Eq. (6) is considered with an assumption$(n - 1 < \alpha < n)$.

For the intention of presenting the influence of the FO-PD controller on governing the overshoot of the second-order system given in Eq. (1), the following transfer function of the FO-PD is considered [22]:

$$C_1(s) = K_{p1} + K_{d1} s^{\mu}, \quad (11)$$

where $\mu \in (1,2)$[23].

The classical PD controller, which is presented for the sake of comparison, is formulated as

$$C_2(s) = K_{p2} + K_{d2} s. \quad (12)$$

4.3 Design of the FO-PD Controller

Consider a second-order system as given in Eq. (1) with$\omega_n = 5$ Rad/s. This system is controlled via an FO-PD controller [as given in Eq. (11)] with the controller parameters $k_{p1} = 2$ and $k_{d1} = 0.25$. The step response is evaluated by applying a unit step reference signal to the input of the considered control system. The maximum overshoot is evaluated for different values of ζ and μ in the ranges of [0,1] and $[1, 5]$, respectively. The result of the previous consideration is illustrated in Fig. 2.

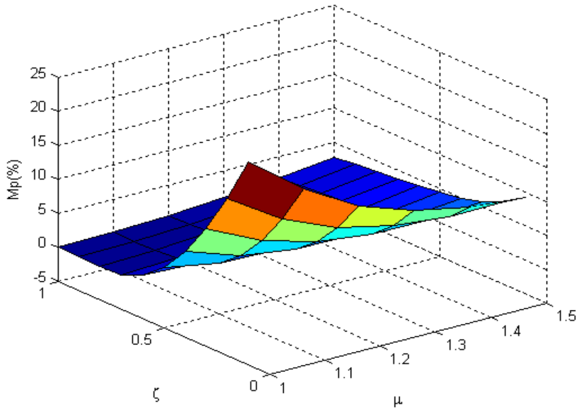

Fig. 2. Maximum percentage overshoot in terms of ζ and μ

MATLAB®'s surface fitting tool (Fig. 3) is applied to create a surface and determine the best fit to the data points of Fig. 2. This tool is utilized to obtain the surface function of the maximum percentage overshoot in ζ and μ for the considered control system. The resulting function of the maximum percentage overshoot is described as the continuous functions of ζ and μ and is described as follows:

$$MP(\%) = c_{00} + c_{10}\mu + c_{01}\zeta + c_{20}\mu^2 + c_{11}\mu\zeta + c_{02}\zeta^2 + c_{21}\mu^2\zeta + c_{12}\mu\zeta^2 + c_{03}\zeta^3. \quad (13)$$

The values of the parameters in Eq. (13) are listed in Table 1.

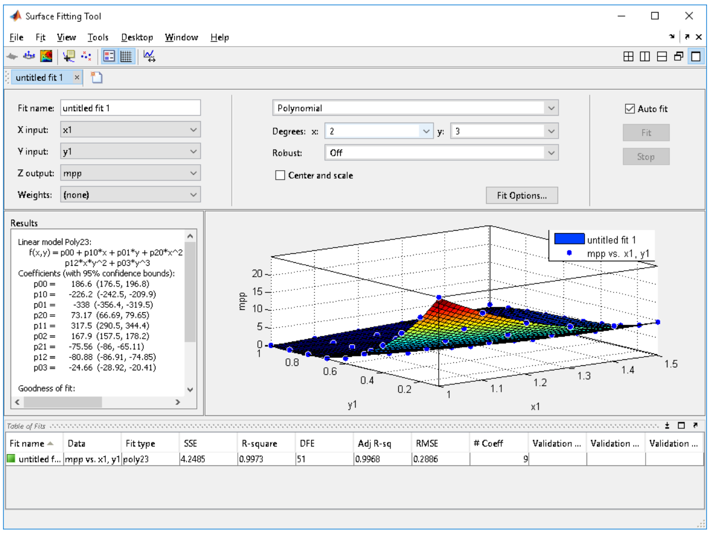

Fig. 3. MATLAB® surface fitting tool

Table 1. Parameters of the obtained surface function given in Eq. (13)

Coefficient	Value
c_{00}	186.6
c_{10}	-226.2
c_{01}	-338
c_{20}	73.17
c_{11}	317.5
c_{02}	167.9
c_{21}	-75.56
c_{12}	-80.88
c_{03}	-24.66

As a verification step, Eq. (13) is plotted for the same value ranges of ζ and μ, and the result is illustrated in Fig. 4. The data points of Fig.s 2 and 4 are compared, and the result is shown in Fig. 5. As illustrated in Fig. 5, the fitting error is about 1%, which allows proceeding to the next step.

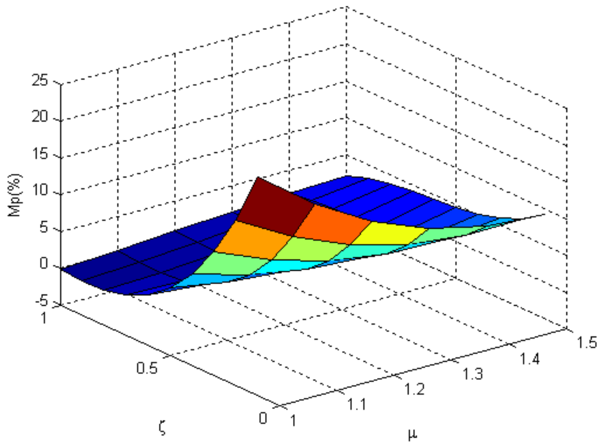

Fig. 4. Verified maximum percentage overshoot surface

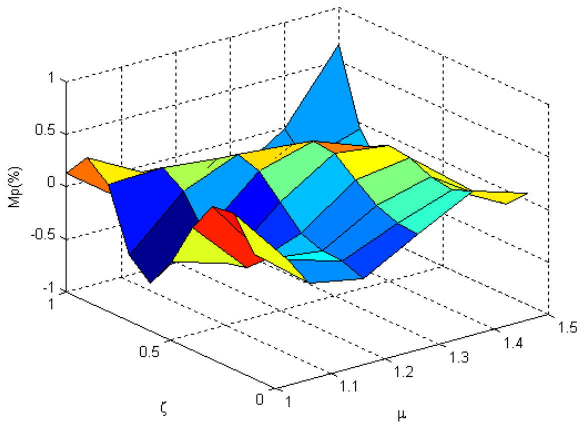

Fig. 5. Error surface results from comparing the original and the verification for the maximum percentage of overshoot surfaces

In Eq. (13), the damping ratio ζ is considered as a design specification and the fractional order is regarded as a design parameter μ. As a result, the following quadratic formula is obtained:

$$k_2\mu^2 + k_1\mu + k_0 - MP(\%) = 0, \tag{14}$$

where the values of the parameters k_0, k_1, and k_2 are expressed as

$$k_0 = c_{00} + c_{01}\zeta + c_{02}\zeta^2 + c_{03}\zeta^3 \tag{15}$$

$$k_1 = c_{10} + + c_{11}\zeta + c_{12}\zeta^2 \tag{16}$$

$$k_2 = c_{20} + c_{21}\zeta. \tag{17}$$

Finally, the quadratic formula given in Eq. (14) is solved for the design parameter μ, design specification ζ, and maximum percentage overshoot $MP(\%)$. The formula is described as

$$\mu = \frac{-k_1 \mp \sqrt{k_1^2 - 4(k_0 - MP)k_2}}{2k_2}. \tag{18}$$

5. Results

Considered Second-order For Wireless Communication Systemsas[24]:

$$G(s) = \frac{b}{s^{2+as+b}}$$

$$s = \frac{-a \pm \sqrt{a^2 - 4b}}{2} = \frac{-a}{2\sqrt{b}}\sqrt{b} \pm \sqrt{b}\sqrt{\left(\frac{a}{2\sqrt{b}}\right)^2 - 1}$$

$$\zeta = \frac{a}{2\sqrt{b}}$$

$$\omega_n = \sqrt{b}$$

$$s = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

Then equation is become:

$$G(s) = \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}$$

The present second order transfer function depending on frequency which effect on consumed power to produce Gp(s) which is represent wireless communication system response. $X(s) + E(s)G(s)G_P(s)$

As a case study, consider a second-order system as given in Eq. (1) with $\omega_n =$ 5Rad/s and$\zeta = 0.1$. This system is controlled via the PD and FO-PD controllers provided by Eqs. (12) and (11), respectively, with controller parameters $k_{p1} = k_{p2} = 2$ and $k_{d1} = k_{d2} = 0.25$. The suitable value of the design parameter μ is required to complete the design of the FO-PD controller, given that the value of the maximum percentage overshoot $MP(\%) = 12$ is based on Eq. (18).

First, the values of parameters k_0, k_1, and k_2 are evaluated using the value of the damping ratio ζ and the constants listed in Table 1. Next, the value of the design parameter \mu is evaluated on the basis of Eq. (18), which is $\mu = 1.28$. Finally, the time response for the conventional PD and FO-PD controllers are established via MATLAB®.

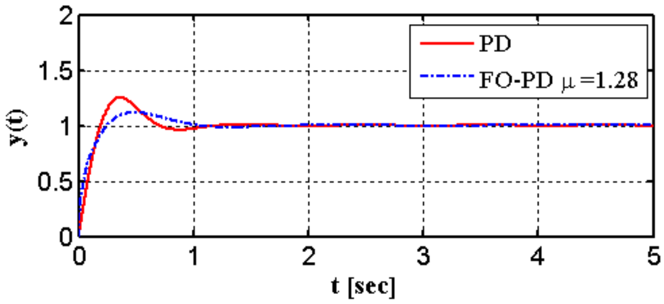

Fig. 6. Step response of the second-order system for $\zeta = 0.5$

Fig. 6 shows that the time-response specifications of the FO-PD controller are better than that of the PD controller in terms of time specifications and maximum percentage overshoot, which is the minimum value, as listed in Table 2. The effect of changing the value of the natural frequency ω_n is shown in Figures 7 and 8, with the percentage maximum overshoots listed in Tables 2 and 3, respectively.

Table 2. Evaluated and maximum percentage overshoot

Controller	MP (%)
PD	22.918
FO-PD	9.953

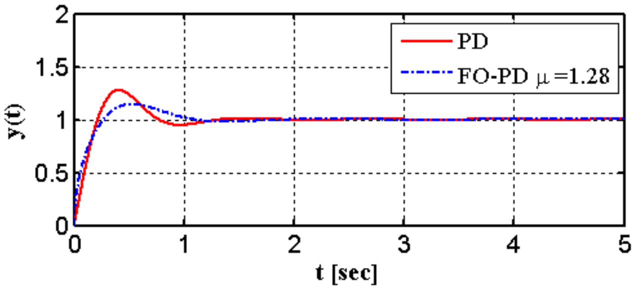

Fig. 7. Step response of the second-order system for $\zeta = 0.5$ and $\omega_n = 4.5$

Table 3. Evaluated and maximum percentage of overshoot ($\omega_n = 4.5$)

Controller	MP (%)
PD	27.832
FO-PD	14.322

Fig. 8. Step response of the second-order system for $\zeta = 0.5$ and $\omega_n = 5.5$

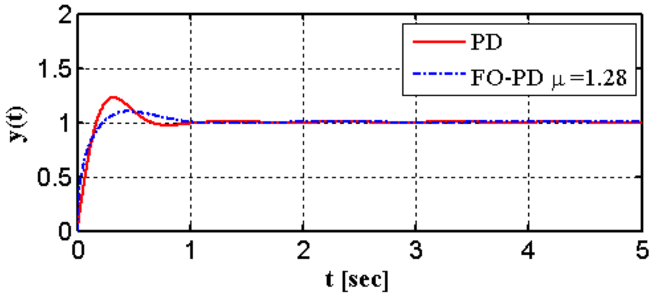

Table 4. Evaluated and maximum percentage of overshoot ($\omega_n = 5.5$)

Controller	MP (%)
PD	25.221
FO-PD	11.902

6. Discussion of results

The FO-PD controller expands and generalizes the conventional PD controller from point to plane. This development enhances the time response by reducing the maximum overshoot and increasing the robustness against parameter variation, as shown in Figures 6 to 8. This advantage results in improved accuracy for control in industrial processes.

7. Conclusion

A simplified second-order model with an FO-PD controller is considered for the wireless networks power control for showing adjust the transmitted power effect on power consume in wireless communication system to solving problems when the service flow routing is determined and an algorithm performance is designed based on the fractional-order control method. The FO-PD controller is represented by three parameters. Two parameters are the same as those of the conventional PD controller. The key point in this research is to simplify the process of obtaining the extra parameter of the controller by respecting the maximum overshoot as a design parameter. FO-PD has one additional parameter that offers further degrees of freedom to the dynamic properties of the controlled model. A comparison of the PD and FO-PD controllers through numerical simulations illustrated that the latter can reduce the maximum percentage overshoot.

References

[1]G. Duan and Q. Hu, "Controllability and observability of second-order time-varying linear systems," in *2015 34th Chinese Control Conference (CCC)*, 2015, pp. 83–87.

[2]P. B. M. Oliveira and D. Vrančić, "Underdamped Second-Order Systems Overshoot Control," *IFAC Proc. Vol.*, vol. 45, no. 3, pp. 518–523, 2012.

[3]D. Gopal, V. Murugesh, and K. Murugesan, "Numerical solution of second-order robot arm control problem using Runge-Kutta-Butcher algorithm," *Int. J. Comput. Math.*, vol. 83, no. 3, pp. 345–356, 2006.

[4]A. M. Singh and Q. P. Ha, "Fast Terminal Sliding Control Application for Second-order Underactuated Systems," *Int. J. Control. Autom. Syst.*, vol. 17, no. 8, pp. 1884–1898, 2019.

[5]Y. Sharma, "DC Shunt Motor Speed Control and analysis using Ziegler-Nichols Algorithm with PID Controller," pp. 89–97, 2015.

[6]D. Valerio and J. S. Costa, "Introduction to single-input, single-output fractional control," *IET Control Theory Appl.*, vol. 5, no. 8, pp. 1033–1057, 2011.

[7]V. Ranjan, R. Nerul, N. Mumbai, S. Jadhav, and M. D. Patil, "Design of Integer and Fractional order PID Controller using Dominant Pole Placement Method," *Int. J. Comput. Appl.*, pp. 975–8887, 2014.

[8]P. Shah and S. Agashe, "Experimental analysis of fractional PID controller parameters on time domain specifications," *Prog. Fract. Differ. Appl.*, vol. 3, no. 2, pp. 141–154, 2017.

[9]D. Sain, S. K. Swain, and S. K. Mishra, "TID and I-TD controller design for magnetic levitation system using genetic algorithm," *Perspect. Sci.*, vol. 8, pp. 370–373, 2016.

[10] R. Matusu, "Matlab toolboxes for fractional order control: An overview," *Ann. DAAAM Proc. Int. DAAAM Symp.*, vol. 22, no. 1, pp. 365–366, 2011.

[11] S. Das, I. Pan, S. Das, and A. Gupta, "Improved model reduction and tuning of fractional-order PIλDμ controllers for analytical rule extraction with genetic programming," *ISA Trans.*, vol. 51, no. 2, pp. 237–261, 2012.

[12] A. K. Maurya, M. R. Bongulwar, and B. M. Patre, "Tuning of fractional order PID controller for higher order process based on ITAE minimization," in *2015 Annual IEEE India Conference (INDICON)*, 2015, pp. 1–5.

[13] A. A. Kesarkar and N. Selvaganesan, "Tuning of optimal fractional-order PID controller using an artificial bee colony algorithm," *Syst. Sci. Control Eng.*, vol. 3, no. 1, pp. 99–105, 2015.

[14] B. Hekimo?lu, "Optimal Tuning of Fractional Order PID Controller for DC Motor Speed Control via Chaotic Atom Search Optimization Algorithm," *IEEE Access*, vol. 7,

pp. 38100–38114, 2019.

[15] B. V. Arun M.K., Biju U., Rajagopal N.N., "Optimal Tuning of Fractional-Order PID Controller.," in *Proceedings of the International Conference on Soft Computing Systems*, 2016.

[16] F. N. Deniz, A. Yüce, N. Tan, and D. P. Atherton, "Tuning of Fractional Order PID Controllers Based on Integral Performance Criteria Using Fourier Series Method," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 8561–8566, 2017.

[17] M. Carlos-Mancilla, E. López-Mellado, and M. Siller, "Wireless sensor networks formation: Approaches and techniques," *J. Sensors*, vol. 2016, 2016.

[18] J. Lorincz, A. Capone, and J. Wu, "Greener, Energy-Efficient and Sustainable Networks: State-Of-The-Art and New Trends," *Sensors 2019, Vol. 19, Page 4864*, vol. 19, no. 22, p. 4864, Nov. 2019.

[19] S. Hasan and A. M. Ismael, "Implementable Self-Learning PID Controller Using Least Mean Square Adaptive Algorithm," *Iraqi J. Sci.*, vol. 2021, pp. 148–154, Jan. 2021.

[20] F. N. Deniz, B. B. Alagoz, and N. Tan, "PID controller design based on second order model approximation by using stability boundary locus fitting," *2015 9th Int. Conf. Electr. Electron. Eng.*, pp. 827–831, 2015.

[21] Y. Yamakawa, Y. Okada, T. Yamazaki, and S. Kurosu, "Tuning Method of PI Controller with Desired Damping Coefficient for a First-order Lag Plus Deadtime System," *IFAC Proc. Vol.*, vol. 45, no. 3, pp. 578–582, 2012.

[22] Ying Luo and Y. Chen, "Fractional-order [proportional derivative] controller for robust motion control: Tuning procedure and validation," in *2009 American Control Conference*, 2009, pp. 1412–1417.

[23] A. Thomas, S. H. Jose, A. Prasad, and A. Ravindran, "Speed Control of a DC Motor Using Fractional order Proportional Derivative (FOPD) Control," 2014.

[24] Aya Moufak Ismael, "ADAPTIVE POWER CONTROL FOR WIRELESS COMMUNICATION SYSTEMS," Al-Nahrain University.